Indian J. Phys. **67B** (1), 73-78 (1993)

Variation of $K/\sigma T$ with altitude and frequency during wave propagation in the ionosphere

R Misra and P K Das*
P K College, Contai, Midnapur–721 401, West Bengal, India

Received 3 July 1992, accepted 9 November 1992

Abstract : A relation between the altitude (h) and the constants A and B connecting $K/\sigma T$ with frequency f(viz. $K/\sigma T = A + Bf^2$) in our previous paper has been found out for ordinary (O) and extra ordinary (E) modes of wave propagation. These relations were used to compute the ratio of thermal conductivity (K) and the product of electrical conductivity (σ) and temperature (T) at different altitudes and for waves of different frequencies. Thus relation for $K/\sigma T$ contain altitude and wave frequency through the constants A and B

Keywords : Thermal and electrical conductivity, mode of wave propagation.

PACS No. : 94.20.Bl

It is known that the thermal conductivity (K), electrical conductivity (σ) and temperature (T) of ionosphere vary with altitude (h). The ratio $K/\sigma T$ depends upon several parameters [1] which include the frequency (f) of a wave during its propagation through the ionosphere [2]. It was shown that the relation between $K/\sigma T$ and f contains two constants (A and B) for the ordinary (O) and extra-ordinary (E) modes of wave propagation. They are different at different altitudes [2].

The relations between A and B with altitude (h) for each mode of wave propagation (O and E), have been developed. These relations will be used to compute values of $K/\sigma T$ at different altitudes for waves of different frequencies.

The values of A and B from our previous work [2] and $K/\sigma T$ have been presented graphically in Figures 1, 2 and 3.

*To whom all communication should be addressed

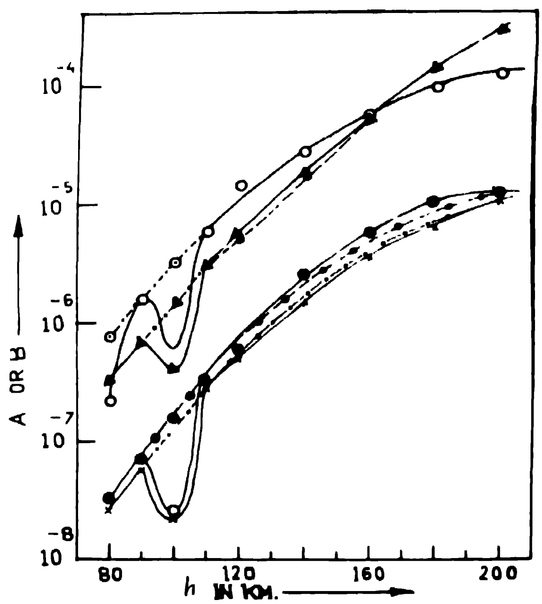

Figure 1. Altitude variation of A and B. Solid curves are drawn by using data from [2] and dotted curves from relations (1) and (2)

For A : O for O-mode and ▲ for E-mode For B ● for O-mode and X for E-mode

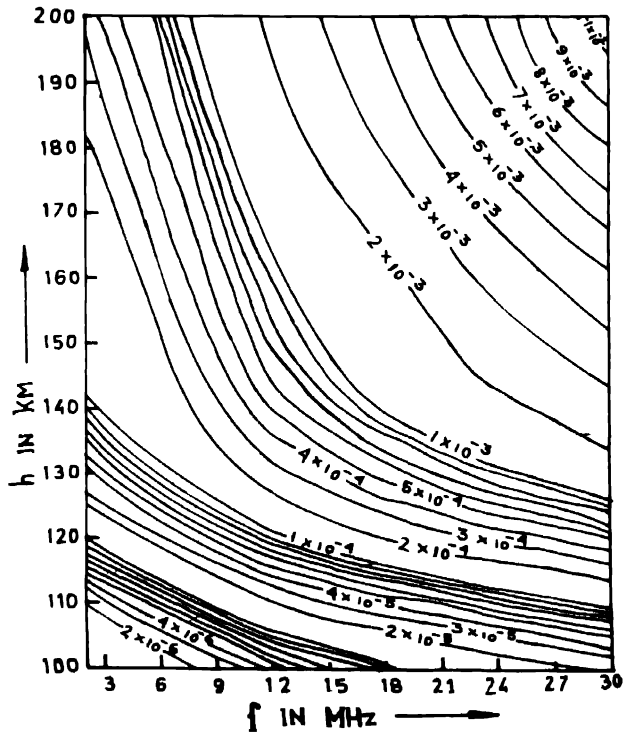

Figure 2. Vaules of $K/\sigma_0 T$ at different altitudes and wave frequencies. Numerals show the

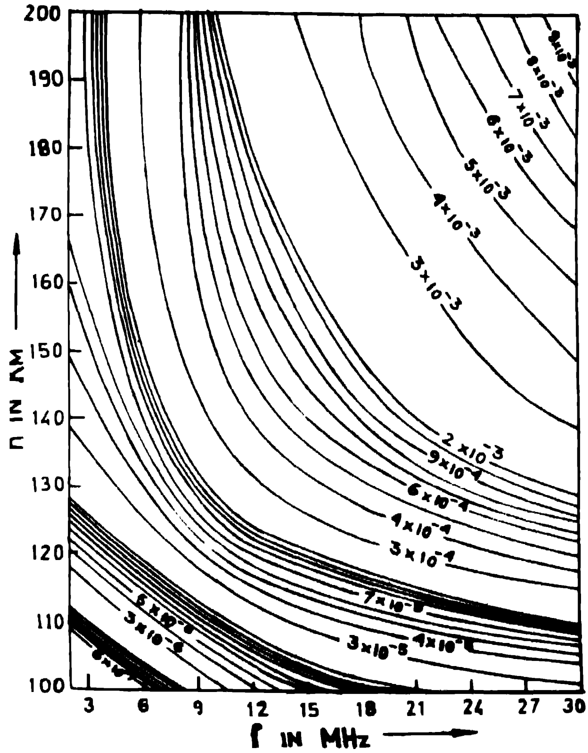

Figure 3. Values of $K/\sigma_t T$ at different altitudes and wave frequencies. Numerals show the values of $K/\sigma_t T$ for the curves on which they appear.

Figure 1 shows that A and B for both modes increase gradually with altitude except near 100 Km, where a depression occurs in agreement with the values of $K/\sigma T$ which, again, depend on the properties of the region at this altitude.

It is to be noted that the values of $K/\sigma T$ computed from these values of A and B do not agree with those computed by using expressions for K [3] and σ [4] below 100 Km and below 6 MH_z. This is evident from Figure 1 and our previous paper [2]. It is because for high electron concentration near 100 Km, the conductivity (σ) becomes high and a depression in $K/\sigma T - h$ curve occurs at this altitude. Hence, the formula proposed in [2] may be applied to wave propagation above 6 MH_z and 100 Km.

From Figure 1 we find that the variation of the logarithms of A and B with altitude for both the O-and E- modes is parabolic except for the depression at about 100 Km. This suggests that the relations between A and B with attitude (h) may be of the forms

$$A = \exp(\alpha_1 + \beta_1 h + \nu_1 h^2) \tag{1}$$

and

$$B = \exp(\alpha_2 + \beta_2 h + \nu_2 h^2) \tag{2}$$

for both O-and E- modes. Here α, β and v's are constants. Using the values of A and B from our previous paper [2] these constants have been computed as shown in Table 1.

Table 1. Values of the constants α, β and υ for O-and E-modes.

Mode of Wave Propagation.	Values of					
	α_1	β_1	υ_1	α_2	β_2	υ_2
O	-22.11	0.1232	-2.879×10^{-4}	-25.8	0.1304	-2.82×10^{-4}
E	-22 132	0.1027	-1.64×10^{4}	-25.83	0.1308	-2.896×10^{-4}

The calculated values of A and B from expressions (1) and (2) using these values of α, β and υ show a slight deviation from those in [2]. They are shown in Figure 1.

It has been observed that A from [2] and those calculated from relation (1) are the same in the O-mode while in the E-mode the values from [2] are slightly lower than those calculated from relation (1). However they become equal above 140 Km. The negative sign has been omitted in A for the E-mode. As far as B is concerned, the values from [2] are higher than those calculated from relation (2) in the O-mode above 120 Km, while in the E-mode they are smaller than the calculated ones upto 140 Km. Above this altitude they become greater but are smaller for 180 Km. In all cases, the difference is small. We wish to point out that the extrapolated portions are in agreement with A and B obtained from relations (1) and (2) respectively in both modes. Values of A and B calculated from relations (1) and (2) are shown by dotted curves in Figure 1.

Finally $K/\sigma\ T$ becomes

$$K/\sigma_0 T = \exp X_1 + (\exp X_2) f^2 \tag{3}$$

$$\text{and } K/\sigma_e T = -\exp X_3 + (\exp X_4) f^2 \tag{4}$$

where σ_0 and σ_e are the electrical conductivities of the region under consideration in O- and E-modes respectively, while

$$X_1 = \exp\ (-22.11 + 0.1232\ h - 2.879\ x\ 10^{-4}\ h^2), \tag{5}$$

$$X_2 = \exp\ (-25.8 + 0.1304\ h - 2.82\ x\ 10^{-4}\ h^2), \tag{6}$$

$$X_3 = \exp\ (-22.132 + 0.1027\ h - 1.64\ x\ 10^{-4}\ h^2), \tag{7}$$

$$\text{and } X_4 = \exp\ (-25.83 + 0.1308\ h - 2.896\ x\ 10^{-4}\ h^2). \tag{8}$$

The relations (3) and (4) provide the variation of $K/\sigma T$ with altitude and frequency of waves simultaneously. Values of $K/\sigma_0 T$ and $K/\sigma_e T$ have been computed with data from ref. [5] which agree well with those calculated using other formulae [3, 4] as in Table 2.

Table 2. Computed values of $K/\sigma T$ for O- and E-modes at different wave frequencies

Altitude h km.	Mode	Values of $K/\sigma T$ for 6 MHz X	6 MHz Y	9 MHz X	9 MHz Y	12 MHz X	12 MHz Y	15 MHz X	15 MHz Y	18 MHz X	18 MHz Y	21 MHz X	21 MHz Y
100	O	1.275 (–6)	1.3 (–6)	2.45 (–6)	2.48 (–6)	4.4 (–6)	4.11 (–6)	6 3 (–6)	6.21 (–6)	8.83 (–6)	8 77 (–6)	1.207 (–5)	1.18 (–5)
	E	5.13 (–7)	3.86 (–7)	1 33 (–6)	1.35 (–6)	2.84 (–6)	2.7 (–6)	4 4 (–6)	4.438 (–6)	6.56 (–6)	6 561 (–6)	9.5 (–6)	9 07 (–6)
120	O	2.84 (–5)	2.89 (–5)	5 44 (–5)	5.62 (–5)	9.8 (–5)	9.45 (–5)	1.4 (–4)	1.43 (–4)	1.96 (–4)	2.039 (–4)	2 62 (–4)	2 75 (–4)
	E	1.14 (–5)	1.124 (–5)	2.95 (–5)	3.254 (–5)	6.3 (–5)	6.224 (–5)	9.8 (–5)	1.006 (–4)	1.46 (–4)	1 47 (–4)	2.03 (–4)	2.02 (–4)
140	O	1.46 (–4)	1.29 (–4)	2.8 (–4)	2.613 (–4)	5.06 (–4)	4.46 (–4)	7.25 (–4)	6.83 (–4)	1.01 (–3)	9.7 (–4)	1 35 (–3)	1.32 (–3)
	E	5 9 (–5)	5.94 (–5)	1.52 (–4)	1 56 (–4)	3 26 (–4)	2 916 (–4)	5.06 (–4)	4.66 (–4)	7 5 (–4)	6.79 (–4)	1.05 (–3)	9.3 (–4)
160	O	2.66 (–4)	2.61 (–4)	5.1 (–4)	5.157 (–4)	9.2 (–4)	8.72 (–4)	1.3 (–3)	1.328 (–3)	1.8 (–3)	1.89 (–3)	2 47 (–3)	2 55 (–3)
	E	1.07 (–4)	1.09 (–4)	2.77 (–4)	3.096 (–4)	5.9 (–4)	5.906 (–4)	9.2 (–4)	9.52 (–4)	1.37 (–3)	1.39 (–3)	1.9 (–3)	1.91 (–3)
180	O	4.5 (–4)	3 44 (–4)	8.6 (–4)	6.5 (–4)	1.55 (–3)	1.1 (–3)	2.2 (–3)	1.65 (–3)	3.1 (–3)	2.3 (–3)	4.14 (–3)	3.14 (–3)
	E	1.8 (–4)	1.736 (–4)	4.68 (–4)	5.67 (–4)	9.98 (–4)	1.11 (–3)	1.55 (–3)	1.805 (–3)	2 3 (–3)	2 66 (–3)	3.21 (–3)	3.66 (–3)
200	O	6.03 (–4)	5.92 (–4)	1 16 (–3)	1 174 (–3)	2 08 (–3)	1 989 (–3)	2.99 (–3)	3.037 (–3)	4.18 (–3)	4.316 (–3)	5.59 (–3)	5.827 (–3)
	E	2.43 (–4)	9.8 (–4)	6.3 (–4)	6.0 (–4)	1.35 (–3)	1.25 (–3)	2.09 (–3)	2.114 (–3)	3.1 (–3)	3 174 (–3)	4.33 (–3)	4.149 (–3)

Table 2. *contd*

24 MHz		27 MHz		30 MHz	
X	Y	X	Y	X	Y
1.52 (–5)	1.53 (–5)	1.84 (–5)	1.92 (–5)	2.3 (–5)	2.37 (–5)
1.21 (–5)	1.2 (–5)	1.56 (–5)	1.58 (–5)	1.94 (–5)	1.89 (–5)
3.36 (–4)	3.57 (–4)	4.21 (–4)	4.5 (–4)	5.14 (–4)	5.54 (–4)
2.69 (–4)	2.66 (–4)	3.45 (–4)	3.39 (–4)	4.3 (–4)	4.2 (–4)
1.74 (–3)	1.712 (–3)	2.17 (–3)	2.16 (–3)	2.66 (–3)	2.661 (–3)
1.39 (–3)	1.22 (–3)	1.78 (–3)	1.55 (–3)	2.22 (–3)	1.92 (–3)
3.16 (–3)	3.314 (–3)	3.96 (–3)	4.179 (–3)	4.84 (–3)	5.146 (–3)
2.53 (–3)	2.52 (–3)	3.24 (–3)	3.2293 (–3)	4.04 (–3)	3.96 (–3)
5.33 (–3)	4.1 (–3)	6.66 (–3)	5.13 (–3)	8.14 (–3)	6.3 (–3)
4.26 (–3)	4.82 (–3)	5.47 (–3)	6.14 (–3)	6.8 (–3)	7.61 (–3)
7.18 (–3)	7.575 (–3)	8.96 (–3)	9.553 (–3)	1.09 (–2)	1.177 (–2)
5.75 (–3)	5.859 (–3)	7.38 (–3)	7.494 (–3)	9.2 (–3)	9.317 (–3)

X-from formulae by [3, 4], Y-from relations (3) and (4)

The relations (3) and (4) may be taken to be valid for ionospheric regions above 100 Km. Variation of $K/\sigma T$ has been presented in Figures 2 (O-mode) and 3 (E-mode). This provides information on its variation with altitude as well as wave frequency.

References

[1] R Misra and P K Das, 1990 *Indian J Phys* **64B** 357

[2] R Misra and P K Das 1991 *Indian J Phys* **65B** 351

[3] R W Schunk 1975 *Planet Space Sci* **23** 437

[4] R G Whitten and I G Poppoff 1965 *Physics of the lower ionosphere* (Englewood Cliffs, N.J., Prentice Hall) P 257

[5] *Physical Research Laboratory Ahmedabad, Data at $F_{10.7}$ = 98.4 at 11 00 A.M., March,* 1987 (Personal Communication)